6 City Dimensions and Efficiency

Introduction

In Chapter 5, we assumed cities are and should be fractal for their function as place of exchange to maximise the efficiency and thus sustainability in terms of transport energy use. This paradigm itself is not an entirely new concept. What makes fractal a useful planning instrument is its fourth and fifth dispositions referred in Chapter 5. These two are, in fact, closely related in the definition of fractal dimension, a measure of the extent to which the city is well developed in some way.

A first practical application of fractal dimension in town planning is introduced in this chapter to express the road network dimension which is, postulated in chapter 5, roughly assumed to contribute to the city's transportation efficiency. This chapter first reviews some theoretical background more deeply why fractal road network is the most efficient in a conventional mathematics of locational optimisation.

There are two most frequently used methods to estimate the fractal dimension: scale method and area methods. In the beginning, fractal geometry has been developed around the scale method, yet the convenience of scale method is used more frequently in urban studies. This chapter seeks to estimate two fractal dimensions in the city: those of population distribution and road network, both of which are based on the area method. These two different dimensions are chosen to show the potential of this new concept, that is, fractal dimension can be applied to any physical object, such as a group of points (population distribution) and lines (road network).

Conventional measurement

In the preceding chapters 4, 5 and 6, we have studied some of the physical factors which affect average journey length to work in the city, especially by introducing fractal dimensions in Chapters 5 and 6. These chapters have isolated each element of the urban structure and sought how it contributes to the journey length. Our interests are, however, not confined just in isolated discussion but also in the relationships between them and the exchange efficiency, which have been identified by many practitioners but have not been discussed in academic fields. The elements of urban structure, such as size, population and density, are not independent. In the same country, larger cities tend to have higher density, but this fact is not comparable between different countries, as the large cities in US have extremely low density, lower than small cities in Europe and Asia. Also, we intuitively assume that these elements affect each other. For example, we usually think large cities tend to have high density and population. It is a fact in England that the largest city, London, has the largest population with the highest density. Gravity model is an approach to verify this assumption, but the study of gravity model has rather sought the application in the practical field based on the assumption. This is therefore an academic interest to study these relations quantitatively to which this chapter is devoted.

Indeed, most urban analyses have so far ignored this fact. In *Beyond Walking Distance*, for example, Manning (1984) discussed travel distances and city size, with a little attention to density. Given the assumption that all the residents working in the city centre, i.e. the so-called single centre model, there is a clear relation between average work journey length and density and between average work journey length and area size (Figure 6.1). This is because the density has decreased and the area has expanded as the time has passed. It is not possible to conclude, therefore, that the density is the determinant factor of the average journey length to work in cities, given that the discussion lacks consideration of size effect. As the conclusive chapter, then, I will seek to reconcstruct these findings in planners'

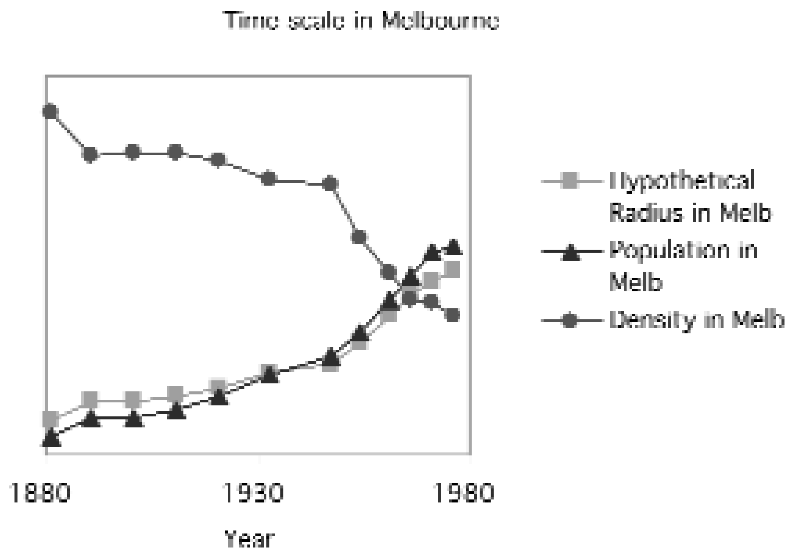


Figure 6.1: Area and Density Shift in Melbourne
Source: modified from Manning (1984)

terms. The approach I will use is bottom-up, rather than planning tradition of top down.

By definition, there is no clear relation between the size $\boldsymbol{A}$ and density $\boldsymbol{r}$ in the city, which are given with population $\boldsymbol{P}$ as:

$$\rho = \boldsymbol{P/A}. \tag{6.1}$$

None the less, we intuitively assume that large cities tend to have higher density. Figure 6.2 shows the relations between size and density and between population and density respectively, which is consistent with our intuitive perception. There seems some relation between area size and density. Negative exponent regression shows its correlation coefficient 0.711 while inverse power regression 0.823. we may assume here the inverse power function for its highest correlation coefficient as:

$$\rho = \kappa \boldsymbol{A}^{-\boldsymbol{d}}. \tag{6.2}$$

From equations (6.1) and (6.2), we obtain an equation:

$$\boldsymbol{P/A} = \kappa \boldsymbol{A}^{-\boldsymbol{d}}, \boldsymbol{P} = \kappa \boldsymbol{A}^{\boldsymbol{d}'}. \tag{6.3}$$

These relations have some implications. First, city's net density, area and population are not

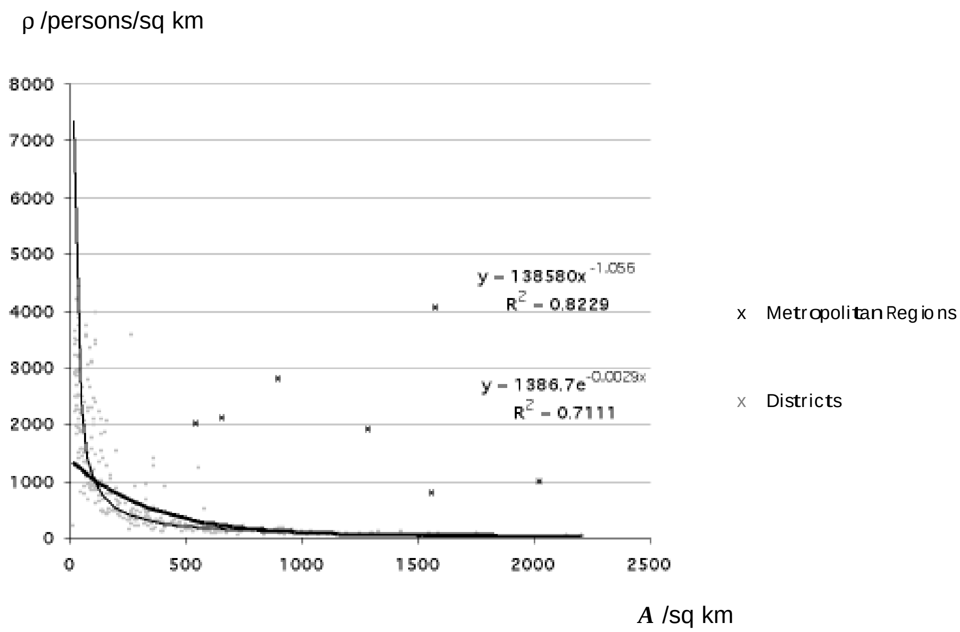


Figure 6.2: City Size and Density

independent but rather deterministic in its geographical and contemporary context. This finding is significant, especially in making policies of city forms. The current policies in the UK, reviewed in Chapter 2, seek high density as a key concept of the compact city. In low-density districts, however, it seems against the law of the natural city as the figure shows that the density is determined largely by its area.

Urban Population Distribution and Fractal Dimension

Some of the early discussions on urban allometry and density, are equivalent to fractal geometry (Batty and Longley 1994). Since the 1940s, it has been identified that, within a adequately self-sufficient city, there is a scaling relation of radius $\boldsymbol{r}$ and population $\boldsymbol{P}(\boldsymbol{r})$ within the radius:

$$\boldsymbol{P}(\boldsymbol{r}) = \phi\, \boldsymbol{r}^{\boldsymbol{D}} \qquad (6.4)$$

$$\rho\,(\boldsymbol{r}) = \boldsymbol{P}\,(\boldsymbol{r}) \,/\, \pi\, \boldsymbol{r}^2 = \boldsymbol{r}^{\boldsymbol{D}-2} \,/\, \pi \qquad (6.5)$$

In the early observations, oddly, $\boldsymbol{D}$ tended to have a value larger than 2, which implies the density to increase with distance, not to decline. Woldenberg (in Batty and Longley 1994), for example, has shown that D varies from around 1.6 to 2.4 depending upon the data set used, and Batty and Longley (1994) see its cause as “researchers have paid very little attention to the definition of area, thus throwing into question the validity of the parameter values estimated, at least in terms of the sorts of theory invoked here' (p.320).

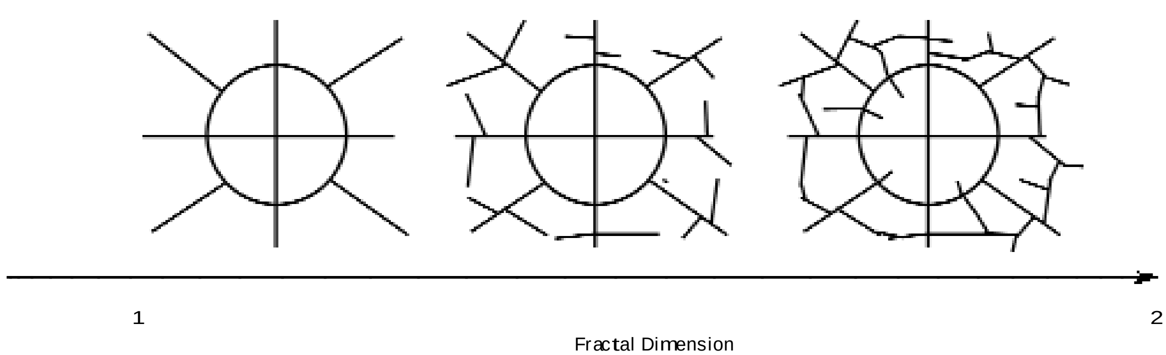
Road Network and Fractal Dimension

When the radius increases, the area ($A(r)$) also increases as $A(r) \sim r^2$. How about the total road length in the circle ($L(r)$)? Empirically,it is known that $L(r)$ increases at the ratio more than r but less than $A(r)$ (e.g. McDonald 1989):

$$L(r) \sim r^D, \qquad (1< D <2) \qquad (6.6)$$

because the roads cover the area with some entities. It shows that, at $D = 1$, the roads are merely lines which connects the city with other cities but do not provide any distribution roads within the city (Figure 6.3). At D=2, on the contrary, the roads cover the area, that is, roads are available evenly in any place of the city. As a result, the city provides alternative roads, which provide alternative place to work (Figure 6.3).

Perhaps, what Murrain (1993) and other gridiron pattern advocates insist is, in this sense, the road network which covers the area with as higher fractal dimension as possible. Fractal



(From left to right) Generating an efficient road network from simple model in a fractal way.
Figure 6.3: Road Network and Fractal Dimension

gives gridiron network a dimension 2 while simplified radial form a dimension merely 1.

Estimation of Road Network Fractal Dimension

Of the most importance is, as has been repeated through the thesis, that it is possible to estimate the dimension of the existing cities and towns. In order to estimate the fractal dimension, there are two types of methods: scale and area methods. A fractal dimension can be estimated by measuring the length (or the number of points, area of objects) at varied scales or areas. Batty and Longley (1994) showed four scale methods. The scale methods require a high resolution map to measure line length over varied scales, and subsequently, enormous time.

The area methods are, on the contrary, less demanding in time and map quality and thus widely used (see Batty and Longley 1994). It is useful particularly when the area is not clearly defined. In this analysis for the discussion of journey length and road network, the census boundary of districts does not define closed city. Rather, we are sometimes interested in a wider region than a district. Here, we take this method to estimate the fractal dimension. Benguigui and Daoud's (1991) estimation method is used, although it is modified because the sample towns are smaller than theirs.

First, the most dense area in road and population is identified as the city's centre. In the case a centre is not such clear, the lowest value of several estimations is used. Next, the total road length $L(r)$ is measure at several radii r. In Benguigui and Daoud's (1991), $L(r)$ is calculated at radius r = 1km, 2km, … , 10km. In the thesis, because of the smaller size of the samples, $L(r)$ is summed at r = 1km, 1.5km, 2km, … , 6.5km. Indeed, within the largest radius 6.5km, the conurbation and geographical factors are subtle, although the range of r still seems larger than the hypothetical radius of the sample cities (Figure 6.4).

This is not actually a new concept, as many studies are equivalent to fractal dimension. In its regression analysis, for example, Wang (1998) found that his population density simulations are best fitted with function:

$$\log \rho(\boldsymbol{r}) = \rho - \gamma \log \boldsymbol{r} \qquad (6.7)$$

Where ρ is the intercept and γ is the slope. In the article, the slope γ varies between 0.4 and 1.7 depending on road density, network pattern (strictly radial and strictly gridiron) and suburban beltways. Taking logarithm of equation (6.7), however, it is clear that:

$$\gamma = 2 - \boldsymbol{D}. \qquad (6.8)$$

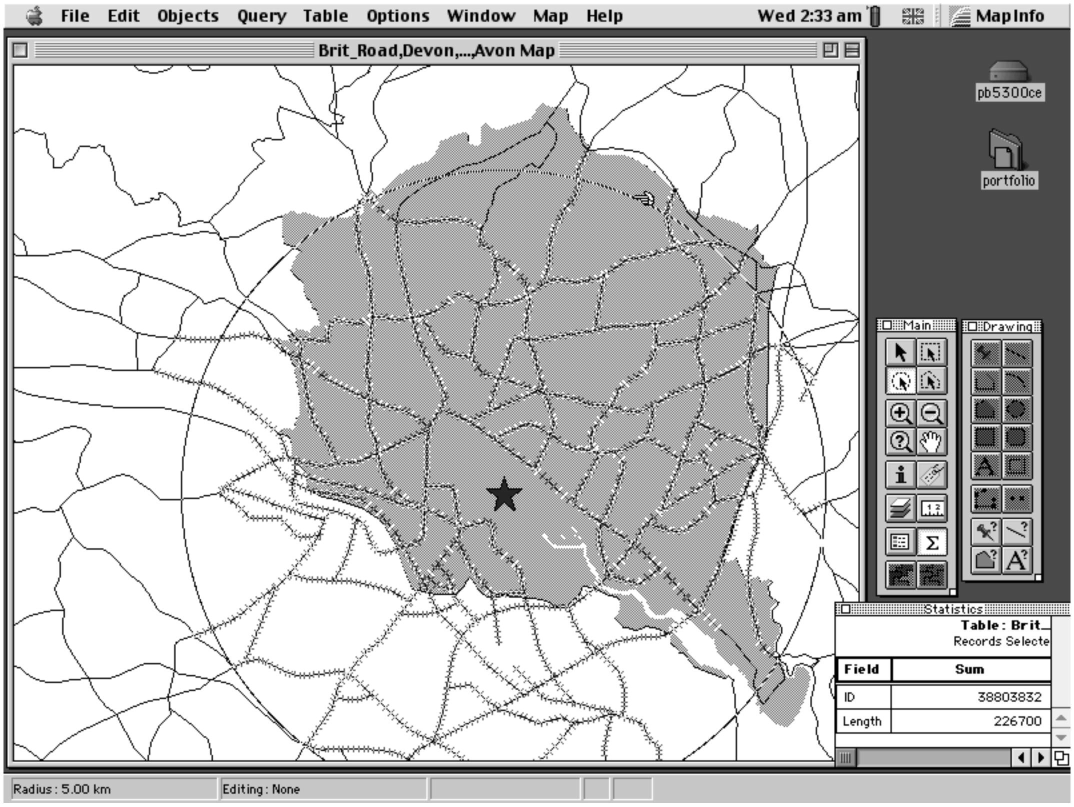
Samples

The samples are all obtained in England to avoid the differences of urban economic, climatic, regional, political, social and other conditions. As reviewed in chapter 4, it is non-sense to compare the cities in the world. In this thesis, therefore, samples are extracted from the districts and the metropolitan areas with at least one town centre. A city may be a district or a metropolitan area. A district may contain non-urban area within.

The road network data are obtained from Bartholomew throughManchester Information Data Archive Service (MIDAS, midas.ac.uk) in ArcView format which are exported to DXF format for analysis on MapInfo. The ArcView formatted data contain road length and other tabular data, which are not available in DXF format. The road lengths are, then, calculated in MapInfo on a workstation. The resulted road lengths are slightly lower than those original in ArcView.

The 1991 Census Digitised Boundary data (UKBORDERS), available from Edinburgh University Data Library (EDiNA) through MIDAS, are used to identify the boundaries of the census districts and metropolitan areas.

The cities which are not self-sufficient imports and exports goods, workers and information



Calculating the total road lengths in the circle on MapInfo
Figure 6.4 : Estimating Fractal Dimension in GIS

more than necessary, which results in longer journey length beyond the city boundary. The extent how the travel length is increased is largely determined by the distances and supply abilities of the nearby cities and, this uncertainty should be avoided. In order to do so, we select the cities with ***S/R*** > 0.800. Density and size are also known to affect the journey length. The samples are selected to have hypothetical radius approximately between 2km and 3km and density 20 persons/hectare and 40 persons/hectare. In this procedure, we gained ten sample cities (Table 6.5). The samples cities also vary in the average journey length to work from 4.70 to 8.29 (excluding home-workers). These cities are, taking account of their high self-sufficiency, small size and high density, well varied in the journey lengths.

The estimation procedure for Plymouth is shown in Figure 6.4, where we have plotted on a double logarithmic scale ln L_r against a function of ln r. The solid line is a linear fit giving an average slope, which is the estimated fractal dimension. The results are show in Table 6.5.

Results and discussion

As intended, the selected samples with as similar a size and density as possible with adequate self-sufficiency. Their areas, however, vary from 38.87 sq km to 96.21 sq km and population from 101,395 to 295,005. Although all the samples have high ***S/R*** as intended and other self-sufficiency indicators does not affect significantly in Chapter 4, the difference in other self-sufficiency parameters may affect the analysis. However, the results are consistent among the cities of this thesis as well as of other studies (Benguigui and Daoud 1991, Batty and Longley 1994). First, the slope is almost consistent throughout the radii measured with correlation coefficients r^2>0.980. Next, the dimension is ranged from 1.4 to 1.9. This means that the roads does not only link to the nearby cities, but also wraps the area to some extent. This range implies how models are different from the reality as the dimensions of Rickaby's six settlement patterns are either less than 1.3 or greater than 1.8 (chapter 5).

As we assumed that the road netowrk dimension affects the average journey length in Chapter 5, ***AJL*** is plotted against the function of ***D*** (Figure 6.6). The samples show a linear relation (r^2=0.658) if Norwich is ignored which showed very small fractal dimension. The result is consistent with our hypothesis that more efficient urban forms take more fine road network at the edges as well as the town centre. In Norwich the smallest travel length despite low road network dimension can be explained to some extent by its smaller area. In addition, although more than 80% of its residents work in the city, not much of its workers (44.9%) live in the city. Indeed, the number of jobs exceeds the number of residents nearly 100% (***R/W*** 55.5%). Approximately 45% of jobs are therefore occupied by those from nearby districts, whose longer journey lengths are not taken into account.

Table 6.5: Fractal Dimension and Average Journey Lengths of Ten Sample Cities

	Area /km²	HR /km	Persons /1	ρ /km⁻²	*R/W*	*S/W*	*S/R*	*D*	*AJL* /km
Norwich	38.87	3.52	212,661	3156	0.555	0.449	0.808	1.15	4.70
Kingston Upon Hull	71.47	4.77	253,111	3541	0.835	0.693	0.830	1.90	5.43
Coventry	96.21	5.53	295,005	3066	0.879	0.702	0.798	1.58	6.47
Derby	77.78	4.98	215,866	2775	0.855	0.707	0.827	1.45	6.45
Exeter	46.89	3.86	101,395	2162	0.742	0.639	0.860	1.83	4.98
Plymouth	79.49	5.03	241,663	3040	0.909	0.829	0.913	1.65	5.74
Leicester	73.08	4.82	272,133	3724	0.674	0.544	0.807	1.63	4.93
Northampton	80.51	5.06	178,570	2218	0.887	0.734	0.828	1.33	8.29
Oxford	45.46	3.80	124,058	2729	0.624	0.521	0.836	1.68	6.35
Stoke-on-Trent	92.44	5.42	244,317	2643	0.842	0.681	0.808	1.73	5.38

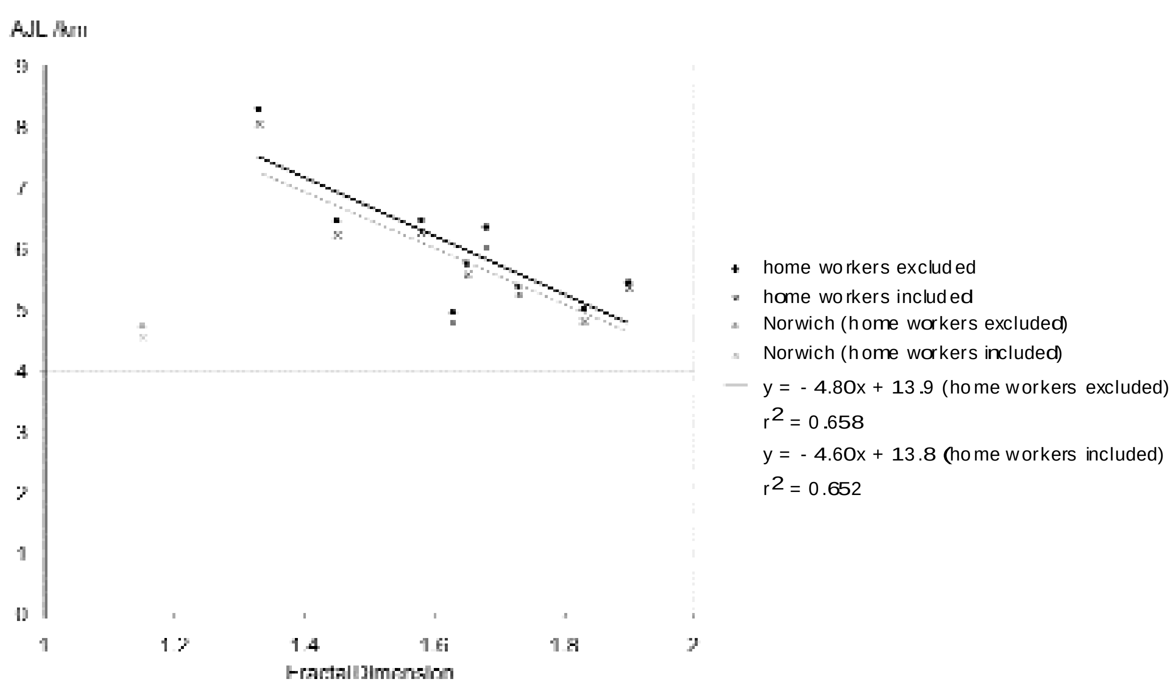


Figure 6.6: Fractal Dimension and Average Journey Length